

2014
AMC 12 A

DO NOT OPEN UNTIL TUESDAY, FEBRUARY 4, 2014

****Administration On An Earlier Date Will Disqualify Your School's Results****

1. All information (Rules and Instructions) needed to administer this exam is contained in the TEACHERS' MANUAL, which is outside of this package. PLEASE READ THE MANUAL BEFORE FEBRUARY 4, 2014. Nothing is needed from inside this package until February 4.
2. Your PRINCIPAL or VICE-PRINCIPAL must verify on the AMC 12 CERTIFICATION FORM (found in the Teachers' Manual) that you followed all rules associated with the conduct of the exam.
3. The Answer Forms must be mailed by trackable mail to the AMC office no later than 24 hours following the exam.
4. *The publication, reproduction or communication of the problems or solutions of this test during the period when students are eligible to participate seriously jeopardizes the integrity of the results. Dissemination at any time via copier, telephone, email, internet or media of any type is a violation of the competition rules.*

The American Mathematics Competitions

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American Mathematics Competitions

65th Annual

AMC 12 A

American Mathematics Contest 12 A

Tuesday, February 4, 2014

INSTRUCTIONS

1. DO NOT OPEN THIS BOOKLET UNTIL YOUR PROCTOR TELLS YOU.
2. This is a twenty-five question multiple choice test. Each question is followed by answers marked A, B, C, D and E. Only one of these is correct.
3. Mark your answer to each problem on the AMC 12 Answer Form with a #2 pencil. Check the blackened circles for accuracy and erase errors and stray marks completely. Only answers properly marked on the answer form will be graded.
4. SCORING: You will receive 6 points for each correct answer, 1.5 points for each problem left unanswered, and 0 points for each incorrect answer.
5. No aids are permitted other than scratch paper, graph paper, rulers, compass, protractors, and erasers. No calculators are allowed. No problems on the test will *require* the use of a calculator.
6. Figures are not necessarily drawn to scale.
7. Before beginning the test, your proctor will ask you to record certain information on the answer form.
8. When your proctor gives the signal, begin working on the problems. You will have **75 minutes** to complete the test.
9. When you finish the exam, *sign your name* in the space provided on the Answer Form.

The Committee on the American Mathematics Competitions (CAMC) reserves the right to re-examine students before deciding whether to grant official status to their scores. The CAMC also reserves the right to disqualify all scores from a school if it is determined that the required security procedures were not followed.

Students who score 100 or above or finish in the top 5% on this AMC 12 will be invited to take the 32nd annual American Invitational Mathematics Examination (AIME) on Thursday, March 13, 2014 or Wednesday, March 26, 2014. More details about the AIME and other information are on the back page of this test booklet.

The publication, reproduction or communication of the problems or solutions of the AMC 10 during the period when students are eligible to participate seriously jeopardizes the integrity of the results. Dissemination via copier, telephone, e-mail, World Wide Web or media of any type during this period is a violation of the competition rules. After the contest period, permission to make copies of problems in paper or electronic form including posting on web-pages for educational use is granted without fee provided that copies are not made or distributed for profit or commercial advantage and that copies bear the copyright notice.

- What is $10 \cdot \left(\frac{1}{2} + \frac{1}{5} + \frac{1}{10}\right)^{-1}$?
 (A) 3 (B) 8 (C) $\frac{25}{2}$ (D) $\frac{170}{3}$ (E) 170
- At the theater children get in for half price. The price for 5 adult tickets and 4 child tickets is \$24.50. How much would 8 adult tickets and 6 child tickets cost?
 (A) \$35 (B) \$38.50 (C) \$40 (D) \$42 (E) \$42.50
- Walking down Jane Street, Ralph passed four houses in a row, each painted a different color. He passed the orange house before the red house, and he passed the blue house before the yellow house. The blue house was not next to the yellow house. How many orderings of the colored houses are possible?
 (A) 2 (B) 3 (C) 4 (D) 5 (E) 6
- Suppose that a cows give b gallons of milk in c days. At this rate, how many gallons of milk will d cows give in e days?
 (A) $\frac{bde}{ac}$ (B) $\frac{ac}{bde}$ (C) $\frac{abde}{c}$ (D) $\frac{bcde}{a}$ (E) $\frac{abc}{de}$
- On an algebra quiz, 10% of the students scored 70 points, 35% scored 80 points, 30% scored 90 points, and the rest scored 100 points. What is the difference between the mean and the median of the students' scores on this quiz?
 (A) 1 (B) 2 (C) 3 (D) 4 (E) 5
- The difference between a two-digit number and the number obtained by reversing its digits is 5 times the sum of the digits of either number. What is the sum of the two-digit number and its reverse?
 (A) 44 (B) 55 (C) 77 (D) 99 (E) 110
- The first three terms of a geometric progression are $\sqrt{3}$, $\sqrt[3]{3}$, and $\sqrt[6]{3}$. What is the fourth term?
 (A) 1 (B) $\sqrt[7]{3}$ (C) $\sqrt[8]{3}$ (D) $\sqrt[9]{3}$ (E) $\sqrt[10]{3}$



American Mathematics Competitions

WRITE TO US!

Correspondence about the problems and solutions for this AMC 10 and orders for publications should be addressed to:

American Mathematics Competitions

1740 Vine Street

Lincoln, NE 68508

Phone 402-472-2257 | Fax 402-472-6087 | amcinfo@maa.org

The problems and solutions for this AMC 2 were prepared by the MAA's Committee on the AMC 10 and AMC 12 under the direction of AMC 12 Subcommittee Chair:

Prof. Bernardo M. Abrego

2014 AIME

The 32nd annual AIME will be held on Thursday, March 13, with the alternate on Wednesday, March 26. It is a 15-question, 3-hour, integer-answer exam. You will be invited to participate only if you score 120 or above or finish in the top 2.5% of the AMC 10, or if you score 100 or above or finish in the top 5% of the AMC 12. Top-scoring students on the AMC 10/12/AIME will be selected to take the 43rd Annual USA Mathematical Olympiad (USAMO) on April 29-30, 2014. The best way to prepare for the AIME and USAMO is to study previous exams. Copies may be ordered as indicated below.

PUBLICATIONS

A complete listing of current publications, with ordering instructions, is at our web site: maa.org/math-competitions

24. Let $f_0(x) = x + |x - 100| - |x + 100|$, and for $n \geq 1$, let $f_n(x) = |f_{n-1}(x)| - 1$. For how many values of x is $f_{100}(x) = 0$?

(A) 299 (B) 300 (C) 301 (D) 302 (E) 303

25. The parabola P has focus $(0, 0)$ and goes through the points $(4, 3)$ and $(-4, -3)$. For how many points $(x, y) \in P$ with integer coordinates is it true that $|4x + 3y| \leq 1000$?

(A) 38 (B) 40 (C) 42 (D) 44 (E) 46

8. A customer who intends to purchase an appliance has three coupons, only one of which may be used:

Coupon 1: 10% off the listed price if the listed price is at least \$50

Coupon 2: \$20 off the listed price if the listed price is at least \$100

Coupon 3: 18% off the amount by which the listed price exceeds \$100

For which of the following listed prices will coupon 1 offer a greater price reduction than either coupon 2 or coupon 3?

(A) \$179.95 (B) \$199.95 (C) \$219.95 (D) \$239.95 (E) \$259.95

9. Five positive consecutive integers starting with a have average b . What is the average of 5 consecutive integers that start with b ?

(A) $a + 3$ (B) $a + 4$ (C) $a + 5$ (D) $a + 6$ (E) $a + 7$

10. Three congruent isosceles triangles are constructed with their bases on the sides of an equilateral triangle of side length 1. The sum of the areas of the three isosceles triangles is the same as the area of the equilateral triangle. What is the length of one of the two congruent sides of one of the isosceles triangles?

(A) $\frac{\sqrt{3}}{4}$ (B) $\frac{\sqrt{3}}{3}$ (C) $\frac{2}{3}$ (D) $\frac{\sqrt{2}}{2}$ (E) $\frac{\sqrt{3}}{2}$

11. David drives from his home to the airport to catch a flight. He drives 35 miles in the first hour, but realizes that he will be 1 hour late if he continues at this speed. He increases his speed by 15 miles per hour for the rest of the way to the airport and arrives 30 minutes early. How many miles is the airport from his home?

(A) 140 (B) 175 (C) 210 (D) 245 (E) 280

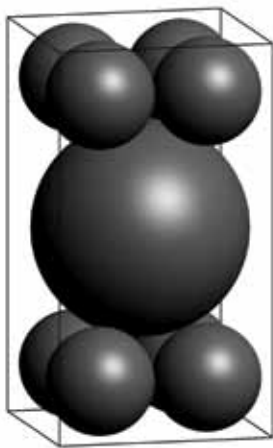
12. Two circles intersect at points A and B . The minor arcs AB measure 30° on one circle and 60° on the other circle. What is the ratio of the area of the larger circle to the area of the smaller circle?

(A) 2 (B) $1 + \sqrt{3}$ (C) 3 (D) $2 + \sqrt{3}$ (E) 4

13. A fancy bed and breakfast inn has 5 rooms, each with a distinctive color-coded decor. One day 5 friends arrive to spend the night. There are no other guests that night. The friends can room in any combination they wish, but with no more than 2 friends per room. In how many ways can the innkeeper assign the guests to the rooms?

(A) 2100 (B) 2220 (C) 3000 (D) 3120 (E) 3125

14. Let $a < b < c$ be three integers such that a, b, c is an arithmetic progression and a, c, b is a geometric progression. What is the smallest possible value for c ?
- (A) -2 (B) 1 (C) 2 (D) 4 (E) 6
15. A five-digit palindrome is a positive integer with respective digits $abcba$, where a is not zero. Let S be the sum of all five-digit palindromes. What is the sum of the digits of S ?
- (A) 9 (B) 18 (C) 27 (D) 36 (E) 45
16. The product $(8)(888\dots 8)$, where the second factor has k digits, is an integer whose digits have a sum of 1000. What is k ?
- (A) 901 (B) 911 (C) 919 (D) 991 (E) 999
17. A $4 \times 4 \times h$ rectangular box contains a sphere of radius 2 and eight smaller spheres of radius 1. The smaller spheres are each tangent to three sides of the box, and the larger sphere is tangent to each of the smaller spheres. What is h ?



- (A) $2 + 2\sqrt{7}$ (B) $3 + 2\sqrt{5}$ (C) $4 + 2\sqrt{7}$ (D) $4\sqrt{5}$ (E) $4\sqrt{7}$

18. The domain of the function $f(x) = \log_{\frac{1}{2}}(\log_4(\log_{\frac{1}{4}}(\log_{16}(\log_{\frac{1}{16}}(x))))))$ is an interval of length $\frac{m}{n}$, where m and n are relatively prime positive integers. What is $m + n$?
- (A) 19 (B) 31 (C) 271 (D) 319 (E) 511
19. There are exactly N distinct rational numbers k such that $|k| < 200$ and

$$5x^2 + kx + 12 = 0$$

has at least one integer solution for x . What is N ?

- (A) 6 (B) 12 (C) 24 (D) 48 (E) 78
20. In $\triangle BAC$, $\angle BAC = 40^\circ$, $AB = 10$, and $AC = 6$. Points D and E lie on \overline{AB} and \overline{AC} , respectively. What is the minimum possible value of $BE + DE + CD$?
- (A) $6\sqrt{3} + 3$ (B) $\frac{27}{2}$ (C) $8\sqrt{3}$ (D) 14 (E) $3\sqrt{3} + 9$
21. For every real number x , let $\lfloor x \rfloor$ denote the greatest integer not exceeding x , and let

$$f(x) = \lfloor x \rfloor (2014^{x - \lfloor x \rfloor} - 1).$$

The set of all numbers x such that $1 \leq x < 2014$ and $f(x) \leq 1$ is a union of disjoint intervals. What is the sum of the lengths of those intervals?

- (A) 1 (B) $\frac{\log 2015}{\log 2014}$ (C) $\frac{\log 2014}{\log 2013}$ (D) $\frac{2014}{2013}$ (E) $2014^{\frac{1}{2014}}$
22. The number 5^{867} is between 2^{2013} and 2^{2014} . How many pairs of integers (m, n) are there such that $1 \leq m \leq 2012$ and

$$5^n < 2^m < 2^{m+2} < 5^{n+1}?$$

- (A) 278 (B) 279 (C) 280 (D) 281 (E) 282
23. The fraction

$$\frac{1}{99^2} = 0.\overline{b_{n-1}b_{n-2}\dots b_2b_1b_0},$$

where n is the length of the period of the repeating decimal expansion. What is the sum $b_0 + b_1 + \dots + b_{n-1}$?

- (A) 874 (B) 883 (C) 887 (D) 891 (E) 892