2020
INTERNATIONAL
MATH CONTEST

Practice Problems for Russian Math Olympiad

Grade 5-6

2019
INTERNATIONAL
MATH CONTEST

ID ____________________
Name ____________________
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RSM Affiliation ____________________
Test Location ____________________

FINAL ROUND

Grades 5-6

RSM FOUNDATION

1

If 20 + *XY* + 19 = 100, compute 20 + *YX* – 19.
(*X* and *Y* represent the digits of the 2-digit numbers *XY* and *YX*.)

2

A car moved 1 second at a constant rate of 2 m/sec, then 1 second at a constant rate of 4 m/sec, then 1 second at a constant rate of 6 m/sec, and so on. All movements were in the same direction. In how many seconds would the total distance covered by the car be 110 meters?

3

Gary and Mary have the same number of candies. If Gary gives Mary half of all his candies, and then Mary gives Gary half of all the candies she has at the moment, Gary would have 12 more candies than Mary. How many candies do Gary and Mary have altogether?

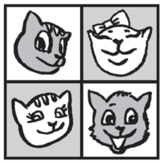
4

Numbers were written in 1000 boxes in a row, one number per box (only the first ten and the last five boxes are shown). For every four boxes in a row, the sum of their numbers was 12. Most of the numbers got erased over time, but three of them remain. What number was written in the last box on the right?

	0					1		2		…					

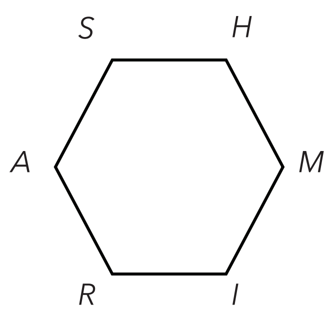
5

Four cats – Astro, Buttons, Calico, and Duchess – bought 30 mice altogether. Each of the four cats bought an odd number of mice. Buttons bought more mice than Astro, and Calico bought fewer mice than Duchess. What is the greatest number of mice that could have been bought by Astro and Calico altogether?



6

The area of a regular hexagon *RASHMI* is 9102 square feet. Compute the area (in square feet) of the triangle *RSM*.



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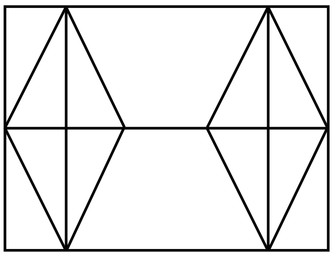
FINAL ROUND

Grades 5-6

RSM FOUNDATION

7 Fatima drew a rectangle with side lengths that were whole numbers. The perimeter of the rectangle was a multiple of 7 and the area was a multiple of 9. Compute the least possible perimeter of Fatima's rectangle.

8 How many quadrilaterals of all sizes and positions are there in the diagram, including quadrilaterals that are made up of more than one shape?



9 Stretch and Shorty are friends. Every January 1st they get measured and they write down the date, Stretch's height, Shorty's height, their total height, and their height difference (the amount by which Stretch is taller than Shorty). From January 1st, 2018, to January 1st, 2019, Stretch grew 5%, Shorty grew 2%, their total height increased by 4%, and their height difference increased by X%. Compute the value of X.

10 How many different whole numbers are there containing only the digits 1 and/or 2 (each of these digits can be used one or more times or not at all) such that for each of these numbers, the sum of all of its digits equals seven?

11 A teacher gave her students a paper square. The first student cut this square into two shapes, using one straight cut not through any of the paper's corners. The second student cut one of the resulting shapes, using one straight cut not through any of that shape's corners, and so on. After ten students had made their cuts, there were eleven shapes, including seven triangles, two quadrilaterals, and a pentagon. How many sides were in the remaining shape?

12 Say that a whole number is "five-important" if it is a multiple of 5 and/or contains the digit 5. For instance, the numbers 55, 120, and 456 are five-important, but the number 2019 is not. How many different five-important numbers are there between 1 and 2019?

Please fold over on line. Write answers on back.

2019 International Math Contest

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Final Round Answers

Problem/Grades	5-6
1	17
2	10
3	48
4	9
5	12
6	4551
7	28
8	52
9	8
10	22
11	10
12	707

2018
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ID ______________________
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FINAL ROUND

Grades 5-6

RSM FOUNDATION

1 Find the second-smallest seven-digit multiple of 11.

2 A bag contains 18 small red, 19 small black, 13 large black, and 15 large red T-shirts, and nothing else. What is the least number of T-shirts Alpa must take out of the bag (without looking at them) to be absolutely sure of having at least two T-shirts among them that differ by both size and color?

3 From a piece of paper Felix cut out some heptagons and octagons with 2018 sides altogether. What is the greatest possible number of octagons among these shapes? (A heptagon is a shape with seven sides.)

4 A fruit drink is made from 25% pure fruit juice and the rest water. A barrel contained some amount of this fruit drink, but then by mistake, 60 liters of water was added to the barrel. How many liters of pure fruit juice must be added to the barrel to correct the mistake, so the barrel would again contain fruit drink with 25% pure fruit juice?

5 Three apples weigh as much as five kiwis. What is the least possible number of apples that weigh more than thirteen kiwis? Assume that all apples weigh the same, and all kiwis weigh the same.

6 Venus took 30 mixed numbers (none of them were whole numbers) and colored each of them either red or blue. On Monday she rounded each red number up and each blue number down to the nearest whole number, added the results, and got 2018. On Tuesday Venus took the same 30 red and blue original numbers, rounded each red number down and each blue number up to the nearest whole number, added the results, and got 2014. On Wednesday she took the same 30 original numbers, rounded each of them down to the nearest whole number, and added the results. How much did she get?

7 How many different positive whole numbers less than 2018 contain only even digits?

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2018
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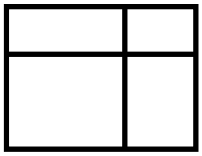
FINAL ROUND

Grades 5-6

RSM FOUNDATION

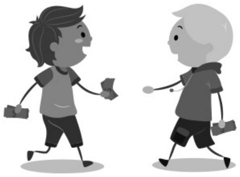
8

A rectangular shape is divided into four non-overlapping rectangular shapes as shown in the diagram. The areas of these four regions (in some order) are 27 square feet, 20 square feet, 18 square feet, and N square feet, where N is a whole number. Compute the value of N.



9

Leo and Theo each had several \$20 and \$50 bills. Leo gave Theo several of his \$20 bills and got from him the same number of \$50 bills, and after this exchange the money was divided equally between the boys. If after that Theo gives Leo all six remaining \$50 bills, each of the boys would have as much money as other one had originally. How many \$20 bills did Leo give Theo?



10

Say that a number is "striped" if all its digits are different, in every even position there is an even digit, and in every odd position there is an odd digit. Assume that the positions are numbered from left to right, starting at 1. For example, the numbers 1 and 1254 are striped, but the numbers 121 and 2018 are not. How many different striped ten-digit numbers are there?

11

Peter was standing in a line of RSM students. There were three times as many students in front of him as behind him. X students (more than 0 but fewer than 30) left the line but Peter and his friend did not. Then three times as many students were behind Peter as in front of him. Compute the sum of all possible different values of X.

12

A factorial number is the product of all whole numbers from one through some whole number. For example, 720 is a factorial number because $720 = 1\times2\times3\times4\times5\times6$. Now let's say that a positive whole number K is "interesting" if some factorial number ends in exactly K zeroes, and "boring" if no such factorial number exists. So the number 1 is interesting since the factorial number 720 ends in exactly one zero. Find the sum of the six smallest boring numbers.

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2018 International Math Contest

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Final Round Answers

Problem/Grades	5-6
1	1000021
2	38
3	247
4	20
5	8
6	2001
7	129
8	30
9	10
10	14400
11	108
12	115

2017
INTERNATIONAL MATH CONTEST

First Name ______________________

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FINAL ROUND

Grades 5-6

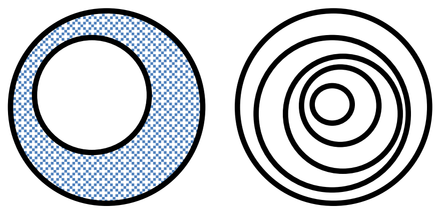
RSM FOUNDATION

1

Today Alice ate 3 fewer candies than yesterday, and twice as many cookies as yesterday. But the total number of candies and cookies she ate today was the same as yesterday. How many cookies did Alice eat today?

2

A ring is a flat shape formed by an inner circle and an outer circle, as shown in the first diagram. How many rings of all sizes and types are there in the second diagram containing five circles?



3

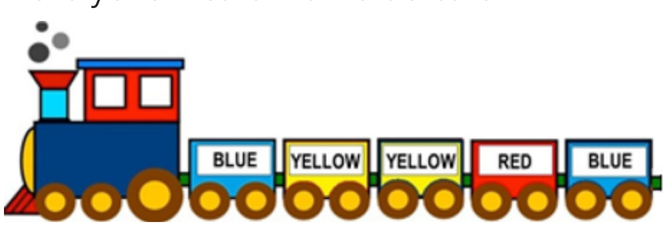
John took two different digits and using them wrote two different 2-digit numbers (each number uses both digits). The sum of these 2-digit numbers is also a 2-digit number. What is the greatest possible value of the smaller of the written numbers?

4

A metal letter R weighs 2 pounds, a metal letter S weighs 1 pound, and a metal letter M weighs 4 pounds. A certain 10-letter “word” containing only the metal letters R, S, and M can be split into three groups of letters weighing 9 pounds per group. How many letters M are in this word?

5

In a very long toy train, the first and last cars were blue. After each blue car (except the last one), there were two yellow cars. After each pair of yellow cars, there was a red car. After each red car, there was a blue car. The first five train cars are shown in the picture. Oleg picked a car and recolored all cars in front of it green. Then Joyce picked a car and recolored all cars behind it green. What is the greatest possible number of non-green cars in the recolored toy train if it contains 7 more yellow cars than blue cars?



6

Find the largest 6-digit multiple of 11 such that the sum of all its digits equals 40.

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FINAL ROUND

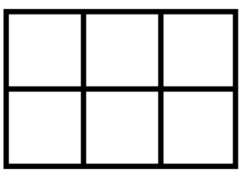
Grades 5-6

RSM FOUNDATION

7 The Treasury wants to reduce usage of coins and is considering several silly proposals: devalue a quarter by X% compared to a penny, raise a dollar by X% compared to a penny, and make a dollar worth 6 quarters. If it decides to implement all three proposals, without any other explicit changes (so a penny is still worth 1 cent, and a dime is still worth 10 cents), how many dimes would be in a dollar?

8 Anna really likes numbers and decided to collect them. She started her collection from the number 1001, which was a birthday gift from RSM. After that, every week Anna added one more new number to the collection by selecting the smallest counting number not yet in the collection that was relatively prime to all the numbers already in the collection. What number was added to Anna's collection on week 10? Note that after 10 weeks the collection contained 11 different numbers.

9 How many different ways are there to place six different digits from 1 to 6 inside the six square cells of a 2-by-3 grid (one digit per cell) such that for every pair of consecutive digits (such as 3 and 4) their square cells share a side?



10 All possible diagonals drawn from the two adjacent vertices A and B of a regular hectogon divide the hectogon's interior into a number of non-overlapping shapes – triangles and quadrilaterals (without any part of a line inside them). How many of these shapes are triangles? (A hectogon is a polygon with 100 sides.)

11 There are eight different cards (four red and four blue) with the digits 2, 0, 1, 7 on them. Each card has exactly one digit, and each of these digits is on exactly two cards (one red and one blue). How many different ways are there to put all eight cards in a row with digits face up and right-side up such that every card appears right next to another card with the same digit?

12 A teacher asked her students to find a 2-digit whole number which has as many as possible different positive factors. Jen realized that there was more than one such number, and listed each of them once. Find the sum of all of Jen's numbers.

Please fold over on line. Write answers on back.

Answers:

Problem No.	Answer
1	6
2	10
3	45
4	5
5	34
6	999922
7	12
8	37
9	16
10	292
11	384
12	402

2016

INTERNATIONAL MATH CONTEST

First Name ______________________

Last Name ______________________

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FINAL ROUND

Grades 5-6

RSM FOUNDATION

1

Jane's mother left some cherries for her children. Jane ate 10 cherries, which was exactly $\frac{2}{5}$ of all the cherries that her mother left. Her brother Sam ate all the remaining cherries. How many cherries did he eat?

2

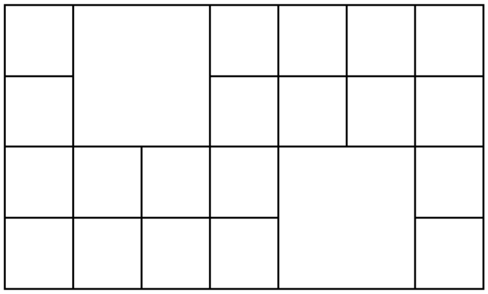
From a big piece of paper Steve cut out 2016 shapes – squares and regular pentagons. Then Michael cut each pentagon along one of its diagonals. How many quadrilaterals were there at the end? (A regular pentagon has five equal sides and five equal angles. A diagonal of a pentagon is a segment which connects two corners that are not already connected by a side.)

3

There are 30 puppies, kittens, and mice altogether in the RSM Pet Hotel. There are twice as many kittens' ears as puppies' tails. There are twice as many puppies' paws as mice's eyes. How many kittens are there in the RSM Pet Hotel (if every animal has the usual number of body parts)?

4

The diagram shows a 4-by-7 rectangle composed of unit squares, where parts of some lines have been erased. How many squares of all sizes and positions are there in this diagram, including squares that are made up of other squares?



5

Stan bought several pizza pies. He cut the first pie into 2 slices, the second pie into 3 slices, the third pie into 4 slices, and so forth. Then he ate one slice from each pie and counted that only 21 slices were left. How many slices did Stan eat?

6

The RSM Seed Company sells seeds for the Rare Rose, which blooms every 12 years; the Seldom Sunflower, which blooms every 7 years; and the Miracle Magnolia, which blooms every 50 years. If all three plants bloom in 2016, in what year will all three of them bloom again the next time?

Please fold over on line. Write answers on back.

2016

INTERNATIONAL MATH CONTEST

First Name ______________________________

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FINAL ROUND

Grades 5-6

RSM FOUNDATION

7

In 1st grade Bob and Pete were the same height. By 6th grade, Bob grew 20% whereas Pete grew 20 cm. By 11th grade, compared with 6th grade, Pete grew 20% whereas Bob grew 20 cm. By how many centimeters is Pete taller than Bob in 11th grade?

8

On Monday, Matthew folded a paper rectangle once to get another rectangle. On Tuesday, he folded this new rectangle once to get another rectangle. Mattew continued to do so daily until he got (after the fifth folding) a 2 cm-by-3 cm rectangle on Friday. What is the greatest possible perimeter (in centimeters) of the original rectangle?

9

There are four pens (black, blue, red, and green) and four pen caps (blue, blue, red, and green). How many ways are there to put all four caps on all four pens (exactly one cap per pen) with the restriction that pen's and cap's colors should be different for each pen? Note that the two blue caps are identical.

10

Ravi wrote (using white chalk) the number 123,456,789 on the board. Then he wrote (using yellow chalk) the number 20 near every white odd digit on the board, and the number 16 near every white even digit on the board. Then he wrote (using pink chalk) the number 20 near every non-pink odd digit on the board, and the number 16 near every non-pink even digit on the board. Finally, he wrote (using grey chalk) the number 20 near every non-grey odd digit on the board, and the number 16 near every non-grey even digit on the board. How many even digits are on the board now?

11

Ben thought of four different positive numbers. Exactly two of his numbers are multiples of 2, exactly two of his numbers are multiples of 3, and exactly two of his numbers are multiples of 5. What is the least possible value of the sum of the four numbers Ben thought of?

12

Say that a pair of numbers X and Y (X may equal Y) is "special" if their sum and their product have the same units digit. How many different special pairs of two-digit whole numbers are there? Count pairs (X, Y) and (Y, X) as one pair.

Please fold over on line. Write answers on back.

2016 RSM Olympiad 5-6

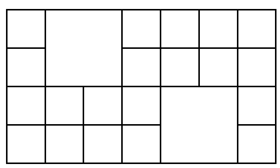
1. Jane's mother left some cherries for her children. Jane ate 10 cherries, which was exactly $\frac{2}{5}$ of all the cherries that her mother left. Her brother Sam ate all the remaining cherries. How many cherries did he eat?
   Answer: 15
   Solution 1. Since the 10 cherries Jane ate were exactly $\frac{2}{5}$ of all the cherries, then 10÷2 = 5 cherries were $\frac{1}{5}$ of all the cherries. Jane's brother Sam ate all the remaining cherries, which was exactly $1 - \frac{2}{5} = \frac{3}{5}$ of all the cherries, so he ate 3×5 = 15 cherries.

   Solution 2. Since the 10 cherries Jane ate were exactly $\frac{2}{5}$ of all the cherries, then Jane's mother left $10 \div \frac{2}{5} = 10 \times \frac{5}{2} = 25$ cherries for her children. So Sam ate 25 – 10 = 15 remaining cherries.

2. From a big piece of paper Steve cut out 2016 shapes – squares and regular pentagons. Then Michael cut each pentagon along one of its diagonals. How many quadrilaterals were there at the end? (A regular pentagon has five equal sides and five equal angles. A diagonal of a pentagon is a segment which connects two corners that are not already connected by a side.)
   Answer: 2016
   Solution. Cutting a regular pentagon along one of its diagonals leaves one triangle and one quadrilateral. Squares are also quadrilaterals. Thus there will be one quadrilateral at the end for each shape Steve cut out, or 2016, no matter how many of these shapes were squares or regular pentagons.

3. There are 30 puppies, kittens, and mice altogether in the RSM Pet Hotel. There are twice as many kittens' ears as puppies' tails. There are twice as many puppies' paws as mice's eyes. How many kittens are there in the RSM Pet Hotel (if every animal has the usual number of body parts)?
   Answer: 10
   Solution. Since kittens have two ears each, there are twice as many kittens' ears as kittens. Since puppies have one tail each, there are as many puppies' tails as puppies. So the number of kittens equals half the number of kittens' ears, and therefore the number of kittens equals the number of puppies' tails which equals the number of puppies. Since puppies have four paws each, there are four times as many puppies' paws as puppies. Since mice have two eyes each, there are twice as many mice's eyes as mice. So the number of puppies equals a quarter of the number of puppies' paws, and therefore the number of puppies equals half the number of mice's eyes which equals the number of mice. This means that the RSM Pet Hotel has the same number of puppies, kittens, and mice for a total of 30 tenants. Thus there are 10 (one third of 30) kittens in the RSM Pet Hotel.

4. The diagram shows a 4-by-7 rectangle composed of unit squares, where parts of some lines have been erased. How many squares of all sizes and positions are there in this diagram, including squares that are made up of other squares?



Answer: 34
Solution. The diagram contains only squares that are 1×1, 2×2, 3×3, and 4×4. There are 20 1×1 squares, 8 2×2 squares, 4 3×3 squares, and 2 4×4 squares, for a total of $20 + 8 + 4 + 2 = 34$ squares of all sizes and positions in the diagram.

5. Stan bought several pizza pies. He cut the first pie into 2 slices, the second pie into 3 slices, the third pie into 4 slices, and so forth. Then he ate one slice from each pie and counted that only 21 slices were left. How many slices did Stan eat?
Answer: 6
Solution. After Stan ate one slice from each pie, there remained 1 slice from the first pie, 2 slices from the second pie, 3 slices from the third pie, and so forth. Since $21 = 1 + 2 + 3 + 4 + 5 + 6$, Stan must have bought 6 pizza pies, and therefore he ate 6 slices.

6. The RSM Seed Company sells seeds for the Rare Rose, which blooms every 12 years; the Seldom Sunflower, which blooms every 7 years; and the Miracle Magnolia, which blooms every 50 years. If all three plants bloom in 2016, in what year will all three of them bloom again the next time?
Answer: 4116
Solution. The prime factorization of number 12 is $2\times2\times3$, number 7 is prime, and the prime factorization of number 50 is $2\times5\times5$. Therefore the least common multiple of 12, 7, and 50 is $2\times2\times3\times7\times5\times5 = (3\times7)\times(2\times5)\times(2\times5) = 21\times10\times10 = 2100$. Thus it will take 2100 years until the next time all three plants bloom in the same year, so the answer is $2016 + 2100 = 4116$.

7. In 1st grade Bob and Pete were the same height. By 6th grade, Bob grew 20% whereas Pete grew 20 cm. By 11th grade, compared with 6th grade, Pete grew 20% whereas Bob grew 20 cm. By how many centimeters is Pete taller than Bob in 11th grade?
Answer: 4
Solution. In 6th grade Pete was 20 cm taller than he was in 1st grade. By 11th grade, compared with 6th grade, Pete grew 20%, so in 11th grade he was taller than in 1st grade by a total of 20 cm, plus 20% of his height in 1st grade, plus 20% of 20 cm (which is $0.2\times20 = 4$ cm). In 6th grade Bob was 20% taller than he was in 1st grade. By 11th grade, compared with 6th grade, Bob grew 20 cm, so in 11th grade he was taller than in 1st grade by a total of 20% of his height in 1st grade, plus 20 cm. Since Bob and Pete were the same height in 1st grade, in 11th grade Pete is taller than Bob by 4 cm.

8. On Monday, Matthew folded a paper rectangle once to get another rectangle. On Tuesday, he folded this new rectangle once to get another rectangle. Mattew continued to do so daily until he got (after the fifth folding) a 2 cm-by-3 cm rectangle on Friday. What is the greatest possible perimeter (in centimeters) of the original rectangle?
Answer: 196
Solution. Let's consider only rectangles with vertical and horizontal sides. First, let's consider two rectangles, *A* and *B*, such that rectangle *A* is fully covered by rectangle *B*. We "unfold" the rectangle *A* once (using any of the possible creases) to get rectangle *C*. Then we can use the same crease (or its extension) to "unfold" rectangle *B* once to get rectangle *D*. It is clear that rectangle *D* fully covers rectangle *C*, and therefore the perimeter of rectangle *D* is not less than the perimeter of rectangle *C*.

Let's also designate the "after the fifth folding" 2 cm-by-3 cm rectangle Z as having horizontal sides 3 cm long and vertical sides 2 cm long. Note that there are just two possibilities for the "after the fourth folding" rectangle Y. The first one has a vertical crease along the 2-cm side. In this case the longest possible adjacent side of rectangle Y is 2×3 = 6 cm long (twice the length of the folded side), and all other possible (for this case) rectangles Y are fully covered by this 2 cm-by-6 cm rectangle Y_1. The second possibility has a horizontal crease along the 3-cm side. In this case the longest possible adjacent side of rectangle Y is 2×2= 4 cm long (twice the length of the folded side), and all other possible (for this case) rectangles Y are fully covered by this 4 cm-by-3 cm rectangle Y_2. Since each possible "after the fourth folding" rectangle Y is fully covered either by rectangle Y_1 or by rectangle Y_2, in order to get the original rectangle having the greatest possible perimeter, we can continue to unfold only rectangles Y_1 and Y_2. By applying similar reasoning, we conclude that each possible "after the third folding" rectangle X is fully covered either by 2 cm-by-12 cm rectangle X_1, or by 4 cm-by-6 cm rectangle X_2, or by 8 cm-by-3 cm rectangle X_3, so in order to get the original rectangle having the greatest possible perimeter, we can continue to unfold only rectangles X_1, X_2, and X_3. After applying similar reasoning a few more times, we conclude that each possible original rectangle U is fully covered either by 2 cm-by-96 cm rectangle U_1, or by 4 cm-by-48 cm rectangle U_2, or by 8 cm-by-24 cm rectangle U_3, or by 16 cm-by-12 cm rectangle U_4, or by 32 cm-by-6 cm rectangle U_5, or by 64 cm-by-3 cm rectangle U_6. Thus, to find the greatest possible perimeter of the original rectangle, we can simply compute perimeters of the six rectangles U_1, U_2, U_3, U_4, U_5, and U_6, and take the greatest of their values. These six rectangles have perimeters of 2×(2 + 96) = 196 cm, 2×(4 + 48) = 104 cm, 2×(8 + 24) = 64 cm, 2×(16 + 12) = 56 cm, 2×(32 + 6) = 76 cm, and 2×(64 + 3) = 134 cm, so the answer is 196.

9. There are four pens (black, blue, red, and green) and four pen caps (blue, blue, red, and green). How many ways are there to put all four caps on all four pens (exactly one cap per pen) with the restriction that pen's and cap's colors should be different for each pen? Note that the two blue caps are identical.
   Answer: 4
   Solution. A cap on the blue pen could be either red or green. Let's start from the case when the blue pen is capped by the red cap, and other three pens are yet uncapped. In this case the green cap could be put on any of the two yet-uncapped non-green pens (black or red, 2 possibilities), and the remaining two blue caps must be put on the two yet-uncapped pens (just 1 possibility, since the two blue caps are identical). In total, we counted 2 ways to put all four caps on all four pens (exactly one cap per pen) with the red cap on the blue pen. By symmetry, there are also 2 ways to put all four caps on all four pens (exactly one cap per pen) with the green cap on the blue pen. Since no other cap can be put on the blue pen, there are a total of 2 + 2 = 4 ways to put all four caps on all four pens (exactly one cap per pen) with the restriction that pen's and cap's colors should be different for each pen.

10. Ravi wrote (using white chalk) the number 123,456,789 on the board. Then he wrote (using yellow chalk) the number 20 near every white odd digit on the board, and the number 16 near every white even digit on the board. Then he wrote (using pink chalk) the number 20 near every non-pink odd digit on the board, and the number 16 near every non-pink even digit on the board. Finally, he wrote (using grey chalk) the number 20 near

every non-grey odd digit on the board, and the number 16 near every non-grey even digit on the board. How many even digits are on the board now?

Answer: 162

Solution 1. Ravi wrote (using white chalk) 5 odd (1, 3, 5, 7, 9) and 4 even (2, 4, 6, 8) digits on the board. For each of the 5 (white) odd digits, he wrote the number 20 in yellow near it. Since both digits 2 and 0 are even, Ravi wrote 10 yellow even digits (5 twos and 5 zeroes). For each of the 4 white even digits, he wrote the number 16 in yellow near it. Since 1 is odd and 6 is even, Ravi wrote 4 yellow odd digits (1s) and 4 more yellow even digits (6s). Thus the total number of even digits now is 4 (white even digits from the original number) + 10 (yellow 2s and 0s) + 4 (yellow 6s) = 18. The total number of odd digits now is 5 (white odd digits from the original number) + 4 (yellow 1s) = 9. All of the digits on the board are non-pink.

Then Ravi wrote, all in pink, 9 twos (even), 9 zeroes (even), 18 ones (odd) and 18 sixes (even). This adds 9 + 9 + 18 = 36 more even digits (for a total of 18 + 36 = 54) and 18 more odd digits (for a total of 9 + 18 = 27). All of the digits on the board are non-grey. Finally Ravi wrote, all in grey, 27 twos (even), 27 zeroes (even), 54 ones (odd) and 54 sixes (even). This adds 27 + 27 + 54 = 108 more even digits, bringing the total number of even digits on the board to 54 + 108 = 162.

Solution 2. For each of the 9 white digits on the board, Ravi wrote two yellow digits near it. After this there are 3 times as many digits on the board as white digits, for a total of 3×9 = 27 white/yellow digits. Similarly, there is a total of 3×27 = 81 white/yellow/pink (non-grey) digits on the board. Each of these non-grey digits is either even or odd.

Ravi wrote the number 20 in grey near every non-grey odd digit on the board. Both digits 2 and 0 are even, so each non-grey odd digit “owns” 2 even digits on the board. Then Ravi wrote the number 16 in grey near every non-grey even digit on the board. Only one of the digits 1 and 6 is even (6), so each non-grey even digit also “owns” 2 even digits on the board (one grey digit near it and itself). Thus, now there are twice as many even digits on the board as non-grey digits, for a total of 2×81 = 162 even digits.

11. Ben thought of four different positive numbers. Exactly two of his numbers are multiples of 2, exactly two of his numbers are multiples of 3, and exactly two of his numbers are multiples of 5. What is the least possible value of the sum of the four numbers Ben thought of?

Answer: 24

Solution. If Ben thought of the four positive numbers 3, 5, 6 = 2×3, and 10 = 2×5, their sum would be 3 + 5 + 6 + 10 = 24, and exactly two of his numbers are multiples of 2, exactly two of his numbers are multiples of 3, and exactly two of his numbers are multiples of 5. Now let’s prove that if the four numbers Ben thought of satisfy all the conditions of the problem, the sum of these four numbers is at least 24.

Exactly two of the four numbers are multiples of 5. If neither of them is exactly 5, then the sum of these two different positive multiples of 5 is at least 10 + 15 = 25 > 24. If one of them is 5 and the other one is at least 20, then their sum is at least 5 + 20 = 25 > 24. If one of them is 5 and the other one is 15, then both of them are odd, so the other two of the four numbers must be the multiples of 2. The sum of these two different positive multiples of 2 is at least 2 + 4 = 6, so the sum of the four numbers is at least 5 + 15 + 6 = 26 > 24.

Finally, if one of the two multiples of 5 is 5 and the other one is 10, then neither of them is a multiple of 3, so the other two of the four numbers must be the multiples of 3. The sum of these two different positive multiples of 3 is at least 3 + 6 = 9, so the sum of the four numbers is at least 5 + 10 + 9 = 24. Thus, the least possible value of the sum of the four numbers Ben thought of is 24.

12. Say that a pair of numbers X and Y (X may equal Y) is "special" if their sum and their product have the same units digit. How many different special pairs of two-digit whole numbers are there? Count pairs (X, Y) and (Y, X) as one pair.
Answer: 171
Solution. If both whole numbers in a pair are odd, then their sum is even (and therefore has an even units digit) and their product is odd (and therefore has an odd units digit), so the sum and the product have different units digits and the pair cannot be special. If one of the two whole numbers in a pair is even and the other one is odd, then their sum is odd (and therefore has an odd units digit) and their product is even (and therefore has an even units digit), so the sum and the product have different units digits and again the pair cannot be special. Thus, both whole numbers making a special pair must be even. In other words, each of them must have an even units digit. Note that for any two whole numbers, the units digit of their sum and the units digit of their product depend only on the units digits of the numbers themselves.

If one of the two whole numbers in a pair has units digit 0, the product of the two numbers has units digit 0, and therefore for the pair to be special the sum of the two numbers must also have units digit 0, so the other number in the pair must have units digit 0. And vice versa, when both whole numbers have the same units digit 0, their sum and their product have the same units digit (namely 0) and the pair is special. Checking other possibilities for the units digits of two even whole numbers (2 and 2, 2 and 4, 2 and 6, 2 and 8, 4 and 4, 4 and 6, 4 and 8, 6 and 6, 6 and 8, 8 and 8) yields that only cases "2 and 2" and "4 and 8" produce special pairs. When both whole numbers have the same units digit 2, their sum and their product have the same units digit (namely 4) and the pair is special. When one of the two whole numbers has units digits 4 and the other one has units digit 8, their sum and their product have the same units digit (namely 2) and the pair is special.

Now let's count how many different pairs of two-digit whole numbers satisfy the following condition: either both numbers have the same units digit 0; or both numbers have the same units digit 2; or one of the numbers has units digit 4 and the other one has units digit 8. Remember that pairs (X, Y) and (Y, X) should be counted as only one pair. There are 9 two-digit whole numbers with units digit 0 (the tens digit could be any of the 9 non-zero digits). These numbers produce $9\times9 = 81$ pairs, 9 pairs of type (X, X) and $81 - 9 = 72$ pairs of type (X, Y) with $X \neq Y$. But we need to count pairs (X, Y) and (Y, X) as one pair, so we have to count 72 pairs (X, Y) with $X \neq Y$ as just $72\div2 = 36$ different pairs. Pairs (X, X) were already counted once each, so there are 9 + 36 = 45 different pairs of two-digit whole numbers having units digit 0. Similarly, there are 45 different pairs of two-digit whole numbers having units digit 2.

Finally, let's count how many different pairs of two-digit whole numbers satisfy the following condition: one of the numbers has units digit 4 and the other one has units digit 8. To avoid counting duplicate pairs (like (14, 98) and (98, 14)), we can simply assume that the first number in a pair has units digit 4, and the second number has units digit 8. There are 9 two-digit whole numbers with units digit 4 (the tens digit could be any of the 9 non-zero digits). Similarly, there are 9 two-digit whole numbers with units digit 8. Thus, there are 9×9 = 81 different pairs of two-digit whole numbers with one number in a pair having units digit 4 and the other one having units digit 8. Altogether, there are 45 + 45 + 81 = 171 special pairs of two-digit whole numbers.