

pose classification Protein-ligand binding prediction with SO(3)-equivariant neural networks

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Agenda

- Structure-based neural networks
- Equivariant networks
- Pose classification task
- Results

Structure-based drug design

The Atomwise story



Point-based networks: Atoms and Distances

Message passing neural networks (MPNN) on atomic graphs



Point-based Message Passing

Aka a pairwise convolution

- Update embeddings depend on
 - Source atom embedding
 - Distance between atoms
- Message passing:

 $F_i \sim \sum_j W(d_{ij}) F_j$

 Edges are functions of position with spherical bessel functions basis:

$$W(r) = \sum_n c_n j_0\left(rac{z_{0n}r}{R}
ight)$$



Pairwise convolution introduced in: Schutt, et. al., NeurIPS 2017 Message passing neural networks: Gilmer, et. al., ICML 2017

SO(3)-Equivariant networks

Limitation: only pass pairwise distances How to capture local geometric information?

Promote scalar hidden states to "geometric" internal state (scalar, vector, tensor, ...)

$\bullet \rightarrow (\bullet, \checkmark, \diamondsuit, \ldots)$

How to do this and maintain symmetry of the problem?

Thomas, et. al., https://arxiv.org/abs/1802.08219 BMA, Hy, Kondor, NeurIPS 2019

Equivariant layers

A rotation R of the input gives a fixed and predictable transformation of output

• Transformation is multiplication by a matrix D(R)

$$f(Rx) = D(R)f(x)$$





Group theory

A general framework for equivariant networks

Geometric tensors can be decomposed into "irreducible representations"

- labeled by integer /=0, 1, 2, ...
- *I*=0: scalar (monopole)
- /=1: vector (dipole)
- *I*=2: tensor (quadrupole)

Spherical tensors have size $(2^{*}/+1)$

Standard basis: Spherical harmonics

Transformation properties by a rotation *R* pre-determined by "Wigner-D matrices" D(R) (2*/+1, 2*/+1)

Tensor products reduced using "Clebsch-Gordan decomposition" (linear transformation)

Spherical Harmonics $Y_m^{\ell}(\hat{\mathbf{r}})$

Basis functions for data on a sphere

Angular components of hydrogen wavefunctions

(2 * / + 1) components

Real spherical harmonics ("tesseral harmonics")



Image credit: https://en.wikipedia.org/wiki/File:Spherical_Harmonics_deg3.png

Clebsch-Gordan transformation

$$Y_{\ell_1}(x)\otimes Y_{\ell_2}(x)=C_{\ell_1\ell_2}\left[igoplus_{\ell=|\ell_1-\ell_2|}^{\ell_1+\ell_2}Y_\ell(x)
ight]$$

Example: I=1 with I=1: $3 \otimes 3 \rightarrow 1 \oplus 3 \oplus 5$

- Scalar product (I=0) $v_1 \cdot v_2$
- Vector product (I=1) $v_1 imes v_2$
- Anti-symmetric traceless product (I=2) $(v_1)_i(v_2)_j + (v_1)_j(v_2)_i \frac{\delta_{ij}}{3}(v_1 \cdot v_2)$

Key ingredients: Geometric tensors

Hidden states: Geometric tensors

$$F_i = \left(F_i^{(0)}, F_i^{(1)}, \dots, F_i^{(\ell)}
ight)$$
 $igodots \longrightarrow \left(igodots, igodots, igodots, \dots, igodots, \dots
ight)$



Thomas, et. al., https://arxiv.org/abs/1802.08219



Key ingredients: Message passing (Pairwise convolution in the paper)

Message passing:

• Edge function is now a spherical tensor

 $W(r) o W({f r}) = R^{(\ell)}(r) \cdot Y^\ell(\hat{f r})$

Convolution is Clebsch-Gordan operation

 $\sum_{j} W(r) F_{j}
ightarrow \sum_{j} W(\mathbf{r}_{ij}) \otimes F_{j}$

Multiplication Clebsch-Gordan
Self-interaction included



$$W(\mathbf{r}) = R^{(\ell)}(r) \cdot Y^\ell(\hat{\mathbf{r}})$$

Thomas, et. al., https://arxiv.org/abs/1802.08219

Key ingredients: Normalization non-linearity

Normalization is also a non-linearity:

$$F_i^{(\ell)} \leftarrow \eta \left(|F_i|^{(\ell)}
ight) rac{F_i^{(\ell)}}{|F_i|^{(\ell)}}$$

Where the norm of a spherical tensor is:

$$\left|F
ight|^{(\ell)} = \sqrt{\sum_{m=-\ell}^{\ell} \left|F_m^{(\ell)}
ight|^2}$$

And we use a learnable function

 $\eta(x)$

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TFNs for pose classification

- Start with TFN ingredients
- Remove all receptor atoms > 7A from ligand
- Input embedding labels atom type and ligand/receptor
- All-to-all passing within a cutoff radius



Implementation based upon: https://github.com/FabianFuchsML/se3-transformer-public

TFNs for pose classification - Equiv. block



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Our model

- 48 filters per layer
- I=(0, 1) (scalar and vector only)
- Readout:
 - All scalars (I=0) from all layers concatenated
 - Readout with a single hidden layer of size 256
- Dropout:
 - p=0.2 in pointwise filters
 - p=0.2 in normalization layer
 - p=0.2 in readout



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Ligand pose classification



Good pose/positive example: Docked molecule (yellow) 1.27Å RMSD to crystal structure (cyan)



Bad pose/negative example: Docked molecule (yellow) 6.57Å RMSD to crystal structure (cyan)

- Positive examples (good poses): ligand position <=2.5Å RMSD to the crystal structure (after receptor superposition)
- Negative examples (bad poses): ligand position >4Å RMSD

Docking modes



Self-docking: Each ligand that has been co-crystallized with a given protein is docked to itself.



Cross-docking:

Each ligand that has been co-crystallized with a given protein is docked to every other crystal structure of the protein.

Dataset and pose sampling

- For each cluster, all ligands were docked using CUina, sampling up to 64 poses separated by at least 1Å RMSD
 - CUina: CUDA-powered docking framework, based upon Vina
 - Written by Adrian Morrison, soon to be released open source
- Dataset was prepared from the PDBBind v2019 protein-ligand set (pdbbind-cn.org) clustered by Uniprot and binding site location
- Final dataset was split into train and test sets according to the sequence similarity of the receptors, such that no receptor in the training set as more than 50% sequence identity to any receptor in the test set

Results - Self-docked

Model	AUC	logAUC**	logAUC (per-target)	top-1***	top-5***
Smina baseline			0.226	61%	86%
MPNN Baseline*	0.890	0.327	0.254	56%	85%
Equivariant Network	0.907	0.360	0.283	66%	90%

Smina Baseline: Use only Smina score for ranking
MPNN Baseline: Our best graph-neural-network on this dataset
logAUC: An early enrichment metric, variant of ROC AUC with log-linear scale. logAUC=0 is purely random, logAUC=85.54% is perfect.
top-n: Probability the top-n poses contains a good pose (< 2.5A RMSD)

Results - Cross-Docked

Model	AUC	logAUC**	logAUC (per target)	top-1***	top-5***
Smina baseline			0.194	48%	76%
MPNN Baseline*	0.847	0.269	0.254	50%	80%
Equivariant Network	0.881	0.320	0.262	55%	83%

Smina Baseline: Use only Smina score for ranking
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Summary

Equivariant networks were used to search for good poses

Equivariance allows for collecting local geometric information

Both cross-docking and self-docking problems saw enrichment of top poses over existing baselines

Acknowledgments

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Directional Message Passing

Point-based MPNN ideas

- Update messages depend on
 - Source atom embedding
 - Distance between atoms
- Messages are linear combinations of Radial Basis Functions:

$$e_{ij}^{(a \to b,l)} = \sum_{c} c_n^{(l)} R_n(r_{ij})$$

e.g., radial Bessel functions

$$R_n(r_{ij}) = j_0\left(\frac{z_{0n}r_{ij}}{R}\right)$$



Pairwise convolution

Pairwise Convolution:

$$egin{aligned} F_i &= \sum_j \operatorname{CG}\left(W(\mathbf{r}_{ij})\otimes F_j
ight) \ & W(\mathbf{r}) &= R^{(\ell)}(r)\cdot Y^\ell(\hat{\mathbf{r}}) \end{aligned}$$



Thomas, et. al., https://arxiv.org/abs/1802.08219



Model	AUC	logAUC**	top-1***	top-5***
Smina baseline	0.687	0.129		
Baseline*	0.847	0.269	50%	80%
Equivariant Network	0.881	0.320	55%	83%

Model	AUC-per-target	logAUC-per-target**	top-1***	top-5***
Smina baseline	0.687	0.129		
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