Joint Deconvolution with Hybrid Norm and Its Applications to Flat Cable Images and Pre-migration Data

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Summary

Joint Deconvolution is an effective tool to attenuate the receiver ghost and broadens the spectrum of the image. Though successful results have been reported for acquisitions using variable-depth streamers, great challenges still exist when it is applied to conventional acquisitions using fixed-depth streamers. Artifacts appear around the receiver notch frequencies where signal to noise ratio is poor. In this paper, we showed that using a hybrid norm optimization which combines joint deconvolution with a L1 norm can reduce the artifact caused by joint deconvolution. Not only can this technology be used on post-stack images obtained with flat cable, but also could it be used for premigration gathers.

Introduction

In marine data acquisition, the up-going wavefield is first recorded by the receivers. Later, it is reflected by the water surface and the down-going wavefield is recorded by the receivers after a time delay, called the receiver ghost. The time delay depends on the depth of the receivers and the emerging angles of the up-going wavefield. Similarly, reflection of the source wavefield before its illuminating the target also causes source ghost in the collected seismic data. Existence of ghost results in the loss of energy near certain frequencies, called notch frequencies, to near zero, accompanied by very low signal to noise ratio. It also limits the bandwidth of the final image and impairs its temporal resolution.

This ghost limitation can be attacked by modifying how the acquisition is applied. These new configurations include, but are not limited to, the variable-depth streamer (*Soubaras, 2010; Soubaras, 2012*), which takes advantages of the rich receiver notch diversity due to different receiver depth along the streamer, dual depth streamer (*Posthumus, 1993*), which has two streamers at different depth, and dual sensor streamer (*Carlson et al., 2007*), which is equipped with both hydrophones and geophones. Using these acquisition methods, the lost energy near the notch frequency of one receiver was compensated by other receivers of different depth or different mechanism. The deghosting process, or deconvolution, is then made easier.

However, the majority of conventional marine data has been acquired using fix-depth streamer, or called flat cable, and it will remain popular in the near future because it is economical and the tools for such configuration are abundant and mature. Finding an efficient and effective deghosting algorithm for flat cable then is important and needed in processing legacy data, to enhance signal of low frequency and push useful frequency spectrum beyond the notch frequencies (*Baldock et al, 2012*). Actually, deghosting is not new for seismic processing (*Morley and Claerbout, 1983*; *Javanovich et al., 1983*), but it gets considerable attention today when high quality broadband images are frequently requested by clients. More interest is now paid on development of new methods or modification of traditional methods (*Zhang and Claerbout, 2010; Wang and Peng, 2012; Zhou et al., 2012*).

Joint deconvolution is a receiver deghosting method especially suitable for variable-depth streamers. It uses both the normal image and the mirror image, which was generated using velocity information. The pioneer work by Soubaras already has exhibited great success when it was used for variable-depth streamers (*Soubaras, 2010; Soubaras, 2012*). In this paper we will show it could also be used for conventional fixed-depth streamer data, when combined with L1 norm optimization. We will discuss the causes of artifacts when applying joint convolution to fixed-depth streamer, and how we can minimize them. Two field data examples are given. One is an application on post-stack images; the other is an example on pre-migration gathers.

Joint deconvolution

Since the receiver ghost always arrives later than the primary, Soubaras (*Soubaras 2010*) assumed that the post-stack migrated image, denoted by $d_{norm}(t)$, also called the normal image, is a convolution of a normalized minimum phase ghost operator $g_{norm}(t)$ with the real reflectivity, $r(t)$

$$d_{norm}(t) = g_{norm}(t) * r(t) \tag{1}$$

where $t = 0, \Delta, 2\Delta, \dots, N\Delta$ and Δ is the time sample length. Unfortunately, the above equation has more than one solution. For any given ghost operator, there is always a solution which will make hold true for the equation. Soubaras addressed this by introducing in a mirror image. The mirror image is obtained by flipping the receivers to their mirror positions about the water surface before migration. When the velocity is correct, the ghost is aligned and the primary appears as a precursor. Thus in addition to equation (1), we have an extra equation for the mirror image,

$$\begin{aligned} d_{norm}(t) &= g_{norm}(t) * r(t) \\ d_{mirr}(t) &= g_{mirr}(t) * r(t) \end{aligned} \tag{2}$$

DOI http://dx.doi.org/10.1190/segam2013-1358.1

where $d_{mirr}(t)$ is the mirror image, and $g_{mirr}(t)$ is a normalized maximum phase ghost operator. In terms of matrix, the equation could be rewritten as

$$D_{norm} = G_{norm}R$$
$$D_{mirr} = G_{mirr}R$$

where R, D_{norm}, and D_{mirr} are vectors formed by arranging the elements of $r(t)$, $d_{norm}(t)$, and $d_{mirr}(t)$ in order, respectively, and G_{norm} and G_{mirr} are matrices corresponding to the ghost operators. When G_{norm} and G_{mirr} are known, one can calculate R in the sense of least square method as

$$R = \left(G_{norm}^T G_{norm} + G_{mirr}^T G_{mirr}\right)^{-1}\left(G_{norm}^T D_{norm} + G_{mirr}^T D_{mirr}\right). \quad (3)$$

It minimizes the L2 norm object function

$$J = \left|\left|d_{norm}(t) - g_{norm}(t) * r(t)\right|\right|_{L2}^{2} + ||d_{mirr}(t) - g_{mirr}(t) * r(t)||_{L2}^{2} \quad (4)$$

Challenges for fixed-depth streamer and L1 norm

The strategy works well for variable-depth streamers, but faces challenges for fixed-depth streamers. Different from the variable-depth stream, receivers are horizontal for a fixed-depth streamer, and the receiver notch diversity is poor or absent. In the extreme case, the time delay from the primary to the ghost is nearly the same from one receiver to another. For example, if the time delay happens to be $n\Delta$, the ghost operators will be given by

$$g_{norm}(t) * r(t) = r(t) - r(t - n\Delta)$$
$$g_{mirr}(t) * r(t) = r(t) - r(t + n\Delta) \quad (5)$$

The minimum phase property or maximum phase property are lost, and the solution is not unique anymore. If $r(t)$ is a solution of equation (2), a set of solutions could be found as

$$r(t) + c_0 + \sum_{i=1}^{\infty} c_i^1 \sin\left(\frac{i2\pi t}{n\Delta}\right) + c_i^2 \cos\left(\frac{i2\pi t}{n\Delta}\right) \quad (6)$$

where c_0, c_i^1, and c_i^2 could be any constants. In other words, the amplitude of all receiver notch frequencies, including the zero frequency, could not be determined even with the mirror image.

The extreme case discussed above seldom happens in practice, but it is close to reality for fixed-depth streamers. The conventional fixed-depth streamer does have some receiver notch diversity, but the notch diversity is poor. Energy near the notch frequencies is low, though not zero, and signal to noise ratio is inferior. Instead of loss of the uniqueness of the solution, artifacts caused by joint deconvolution, or other deghosting methods, mostly appear as ringing near the notch frequencies or as low frequency noise.

Mismatching between the deghosting operator and ghost operator is a main culprit of the unwanted artifacts. Taking equation (5) as an example again and assuming the time delay to be T, in frequency domain, the operator could be written as

$$1 - e^{-i\omega T}. \quad (7)$$

If the deghosting algorithm mistaken T as $T + \delta$, the expected deghonsted image will be

$$\frac{1 - e^{-i\omega T}}{1 - e^{-i\omega(T+\delta)}} r(\omega)$$

where $r(\omega)$ is the Fourier transform of $r(t)$. Some simple computation tells us the amplitude of the error is

$$\left|\frac{\sin\left(\frac{\omega\delta}{2}\right)}{\sin\left(\frac{\omega(T+\delta)}{2}\right)} r(\omega)\right|.$$

When $\delta \neq 0$, the error will not be zero and is proportional to the amplitude of the ghost free image. Thus artifacts are more frequently observed in processing deep streamer data. This becomes more serious close to the notch frequencies when the denominator is close to zero. Noise is also amplified near these frequencies, which in turn may also contribute to ringing.

However, mismatching and noise is universal in practice, and only different in its scope. One intuitive solution is to incorporate the energy of $r(t)$ into the optimization with the hope that the over-amplified notch frequency energy can be reduced. If the L2 norm is used, the object function (4) will be change to

$$\begin{aligned} J = \ & \left|\left|d_{norm}(t) - g_{norm}(t) * r(t)\right|\right|_{L2}^{2} \\ & + ||d_{mirr}(t) - g_{mirr}(t) * r(t)||_{L2}^{2} \\ & + \lambda||r(t)||_{L2}^{2} \end{aligned} \quad (8)$$

and λ is a weight factor user can change. The optimization could be easily solved by replacing equation (3) with

$$R = \left(G_{norm}^T G_{norm} + G_{mirr}^T G_{mirr} + \lambda I\right)^{-1}\left(G_{norm}^T D_{norm} + G_{mirr}^T D_{mirr}\right) \quad (9)$$

where I is the unit matrix. This also makes the computation stable by adding a positive constant on the diagonal elements. Equation (9) also shows that the L2 norm optimization leads to a linear filter.

An alternative method is using L1 norm instead of L2 norm. L1 norm of a function is the sum of the absolute values of all elements. The object function is now

$$\begin{aligned} J = \ & \left|\left|d_{norm}(t) - g_{norm}(t) * r(t)\right|\right|_{L2}^{2} \\ & + ||d_{mirr}(t) - g_{mirr}(t) * r(t)||_{L2}^{2} \\ & + \lambda||r(t)||_{L1} \end{aligned} \quad (10)$$

Different from the linear filter given by (9), the solution of an L1 norm optimization is not linear, and this benefits the deghosting. In the ideal situation when equation (2) is accurate, the two optimizations will make little difference. If additional small noise is present in the normal data and mirror data, L1 norm will be more robust since it will still give us the real answer while L2 norm will show some

DOI http://dx.doi.org/10.1190/segam2013-1358.1

error (*Candes 2008*). In practice, equation (2) is just an approximation and noise is strong. With L2 norm, which is the square root of the sum of the square of all elements, larger amplitude is affected more than smaller amplitude. As a tradeoff, one single strong peak is replaced be one less strong peak with several weak peaks if the result normal and mirror data are almost the same. From equation (6), we know that signal near ghost notch frequency will not change $d_{norm}(t)$ and $d_{mirr}(t)$ too much while reducing the norm of $r(t)$.

Thus L1 norm is superior to L2 norm in the sense that it evenly weight strong and weak amplitude, but existing algorithms for L1 norm optimization are too computationally expensive for seismic processing. We find it efficient to use the hybrid norm suggested by Claerbout (*Claerbout, 2009; Zhang and Claerbout, 2010*), which is given by

$$||r||_{hb} = \varepsilon^2(\sqrt{1+\frac{r^2}{\varepsilon^2}}-1) \qquad (11)$$

where ε is a small constant. The object function then changes to

$$\begin{aligned} J = \quad & ||d_{norm}(t) - g_{norm}(t) * r(t)||_{L2}{}^2 \\ & +||d_{mirr}(t) - g_{mirr}(t) * r(t)||_{L2}{}^2. \qquad (12) \\ & +\lambda||r(t)||_{hb} \end{aligned}$$

The norm function given by equation (11) is differentiable, thus we can take advantage of all the gradient based optimization methods.

Applications to pre-migration or pre-stack gathers

In addition to the post-stack image, the same algorithm could also be applied for common offset gathers before migration or after migration but before stack. When used in the pre-stack mode, it is similar to that proposed by Soubaras (*Soubaras 2012*). But instead of working in a CDP gather and using constraints on the ghost-free reflectivity gather to stabilize the computation, this algorithm works on a common offset gather and uses neighboring traces to make the computation stable.

Also similar to Soubaras' method, a set of mirror data is required in the computation. For pre-stack mode, the mirror data is obtained from a mirror migration. For pre-migration gathers, the mirror data is obtained by redatuming the recorded data from the mirror position of receivers to their normal positions and flipping the polarity. The normal data and the mirror data then are processed jointly to derive the ghost-free data set.

Wang and Peng also proposed a pre-migration deghosting method in the F-XY domain (*Wang and Peng, 2012*), which is very promising. Different from our algorithm, Wang and Peng's algorithm works in the frequency domain and assumes the ghost operator in the form of equation (7). In this way it only has to find delay time T and a mismatching parameter between the mirror data and normal data, instead of two ghost operators in time domain. But increasing the number of unknown parameters does not increase the cost of our algorithm significantly, since solving ghost operators only takes a fraction of the computation time compared with the time used to solving the image. More than that, using more parameters for the ghost operators leaves the algorithm more space to tolerate the variations of ghost operators within a computation window, makes it more stable in complex situations.

Field data example on post-stack image

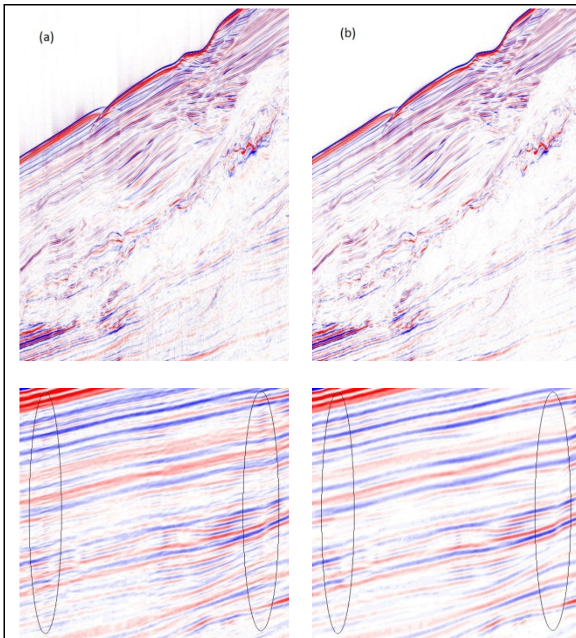

Figure 1: Images obtained using joint deconvolution on post-stack images (top) and an elarged section (bottom) for comparing L2 norm and L1 norm. (a): image obtained after joint deconvolution using L2 norm penalty on $r(t)$; (b): image obtained after joint deconvolution using L1 norm on $r(t)$. Ghost operators are the same in both computations, and all figures use the same amplitude scale.

We tested joint deconvolution with L1 norm on two sets of field data. One was a post-stack image; the other is pre-migration common offset gathers.

Figure 1 shows the comparison of L1 and L2 norm optimization on a post-stack image. The data was acquired offshore Gambia with fixed depth streamer of depth 9m. Time Kirchhoff migration was used to obtain the normal image and mirror image. We first ran a joint deconvolution with L2 norm on the stacked normal and mirror images.

DOI http://dx.doi.org/10.1190/segam2013-1358.1

The result could be found in figure 1a. Although the receiver ghost has been attenuated effectively, when compared with the normal image (not shown here), obvious low frequency noise and ringing contaminates the image. Another joint deconvolution with L1 norm then was applied using the ghost operators obtained in the previous L2 norm optimization. The result is shown in figure 1b, and the artifacts were greatly reduced. A zoomed in figures were also shown in the figure.

Field data example on pre-migration gathers

Encouraged by the success in post-stack images, we tested the same algorithm on common offset gather before migration. We get several advantages by doing this. Firstly, the deghosted CDP gather has better temporal resolution and helps the following processing including velocity analysis; secondly, it saves the time required to run mirror migration multiple times; and finally, it does not require an accurate velocity. Figure 2 is a field data example, also from offshore Gambia, with receiver depth of 9m. Figure 2a shows a CDP gather before joint deconvolution. We used one dimensional ray-tracing method in redatuming the data to generate the mirror data, which could be seen in figure 2b. Both data sets were then resorted into common offset gathers and deghosted using joint deconvolution. The same CDP gather as in figure 2a and 2b after joint deconvolution was shown in figure 2c. It could be noticed that noise is also attenuated since joint deconvolution tends to depress anything which does not match between the normal data and mirror data, including the noise. The stacked image using normal gathers and gathers after joint deconvolution are displayed in figure 2d and 2e, respectively. The receiver deghosting effect was clearly evident.

Conclusions

Because of the lack of receiver notch diversity, joint deconvolution faces great challenges in processing fixed-depth streamer data. In this paper, we have shown that L1 norm optimization effectively attenuates the artifacts caused by joint deconvolution in processing fixed-depth streamer data. The algorithm was tested for both post-stack image and pre-migration common offset gathers using field fixed-depth streamer data, and the results are successful. We note that it could also be used for variable-depth streamers, either for the post-stack images or the pre-migration data set, although only fixed-depth steamer examples were displayed in this paper.

In a reasonably sized widow for the joint deconvolution the variance of ghost operators can not be neglected. It is impossible, or at lease very difficult, to find one ghost operator that matches perfectly everywhere. As such, in practice, we have to accept the fact that the ghost operator we obtained is not accurate, either because of noise, too sparse sampling, bad migration, or something else. This requires the deconvolution algorithm to be robust and smart. In this sense, L1 norm is superior to L2 norm.

Acknowledgments

The authors wish to thank Simon Baldock, Manhong Guo, Sampad Laha, and Zhaojun Liu for their inspiring discussions and suggestions, Yongbo Zhai for helping generating the mirror data, and Laurie Geiger and Chuck Mason for proofreading and editing this paper. The authors also wish to express their appreciation to TGS for permission to publish this paper.

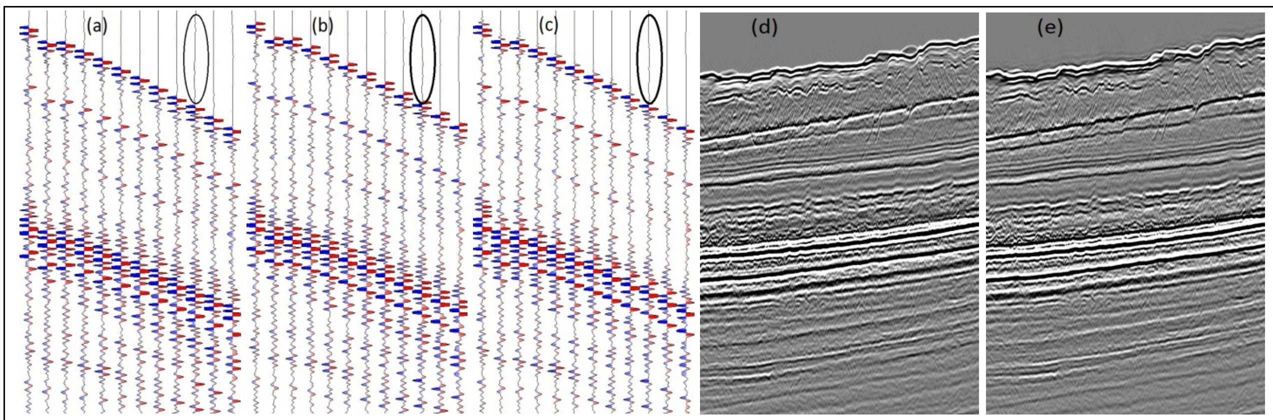

Figure 2: Premigration deghosting using joint deconvolution with L1 norm. (a): section of one CDP gather before joint deconvolution; (b): section of the same mirror CDP gather generated based on one-dimensional ray-tracing; (c): the same CDP gather section after joint deconvolution; (d) stacked image after Kirchhoff migration using the un-deghosted data; (e) stacked image after Kirchhnoff migration using deghosted data. The low frequecy swell noise was reduced by joint deconvolution, see the circled area.

DOI http://dx.doi.org/10.1190/segam2013-1358.1

http://dx.doi.org/10.1190/segam2013-1358.1

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